

Antiprotonic hydrogen in static electric field

G.Ya. Korenman and S.N. Yudin

*Skobeltsyn Institute of Nuclear Physics, Moscow State University,
Moscow, Russia*

e-mail: korenman@anna19.sinp.msu.ru

Abstract

Effects of the static electric field on the splitting and annihilation widths of the levels of antiprotonic hydrogen with a large principal quantum number ($n = 30$) are studied. Non-trivial aspects of the consideration is related with instability of $(p\bar{p})^*$ -atom in ns and np -states due to coupling of these states with the annihilation channels. Properties of the mixed nl -levels are investigated depending on the value of external static electric field. Specific resonance-like dependence of effective annihilation widths on the strength of the field is revealed.

1 Introduction

When antiproton enters a matter it is slowed down and captured eventually by atom forming an antiprotonic atom. From the general point of view this new physical object is of the great interest. For the long time the experimental methods to investigate antiprotonic atoms have been limited by studying their X-ray radiation and the products of antiproton annihilation at the end of cascade transitions. However, about 15 years ago the investigations of these atoms have got on the new qualitative level especially due to the new experimental methods (see [1, 2] and references therein) that used antiprotonic beam from storing ring of LEAR operated in CERN up to 1996. New setup AD ("antiproton decelerator") that has been operating since the end of 2001 opens, in definite aspects, even more possibilities for experimental study of antiprotonic atoms. In particular, in the frame of ASACUSA project [3] it is supposed, among other experiments, direct observation and investigation of antiprotonic atoms formation in very rarefied gases.

The new experimental possibilities pose various problems of the theory of antiprotonic atoms that were not considered early because they seemed to be of little importance in the former experimental conditions. One of such problem could be an influence of external static electric field on the properties of antiprotonic atoms, especially antiprotonic hydrogen. Static electric and magnetic fields can be generated by some parts of an experimental setup, therefore they have to be taken into account in precision measurements of the properties of excited $(p\bar{p})$ atom. On other hand, a special choice of the fields could be used in order to obtain an additional information on the properties of antiprotonic atoms.

According to current concepts of the theory of exotic atom formation (see, e.g., [4, 5]), the antiprotonic atoms are formed initially in highly excited states with principal quantum number $n \geq n_0 \simeq \sqrt{\mu}$ and, predominantly, with large values of orbital angular momenta

($1 \ll l \leq n - 1$), where μ is a reduced mass of the \bar{p} - nucleus system in the units of electron mass (so, for the formation in atomic hydrogen $\mu = 918$, $n_0 = 30$). When $(p\bar{p})_{nl}$ atom is formed in molecular hydrogen, the distribution over quantum number is more complicated, however the probability of population of the states with large n and l remains rather appreciable. Among the highly excited states, specific interest can present circular orbits with $l = n - 1$ and nearly-circular orbits. In isolated antiprotonic hydrogen atom these states can decay only by radiative E1-transition. Radiative life time of hydrogen-like atom at large n, l can be estimated by the equation [6]

$$\tau_\gamma(nl) \simeq (n^3 l^2 / \mu Z^4) \cdot 84.5 \text{ ps} \quad (1)$$

that gives $\tau_\gamma = 2.09 \mu\text{s}$ for the circular orbit with $n = 30$ of antiprotonic hydrogen. This life time is of the same order of value as for antiprotonic helium metastable states [1], therefore the same high-precision method of laser-induced transition could be, in principle, applied for the study of energy structure and dynamics of the excited states of antiprotonic hydrogen. However, the $(p\bar{p})$ atom, as compare with $(\bar{p}\text{He}^+)$, may be less stable against atomic collisions and influence of external fields, because a most part of the $(p\bar{p})_{nl}$ states with a definite n is degenerated in l , contrary to antiprotonic helium. Therefore theoretical study of the quenching effects on the long-lived states of antiprotonic hydrogen is important for the possible future experiments. One of the most important factors to quench long-lived states is the electric field arising in the processes of atomic collisions or as a results of the external conditions. The dynamical (collisional) Stark effect in hadronic hydrogen atoms was discussed in the literature for a long time [6, 7]. However the influence of the external static electrical field, as far as we know, was not discussed in the literature, in spite of the seeming simplicity of this effect. In this paper we discuss the influence of electric field on the splitting and decay rates of highly-excited ($n = 30$) states of antiprotonic hydrogen atoms.

2 Formulation of the problem

The states with large quantum numbers n, l of isolated antiprotonic hydrogen $(p\bar{p})_{nl}$ are long-lived, because annihilation in the states with $l \gg 1$ is practically absent and, on other hand, these states have large radiative life times. However, if some external action forces transitions of the $(p\bar{p})$ system to the states with low orbital angular momenta, then antiproton will annihilate quickly. In the problem under consideration an external action is produced by the static external electric field. Formally this problem is formulated as follows.

If the nuclear $(p\bar{p})$ -interaction is disregarded, all the states nl of antiprotonic hydrogen atom will be degenerated in l . In the real system nuclear interaction is important in the states with low angular momenta and it produces the both shifts and annihilation widths of the levels, or, in other words, complex shifts

$$\Delta E_{nl} = -\epsilon_{nl} - i\Gamma_{nl}/2. \quad (2)$$

The estimations show that these values at $n \sim 30$ are important for $l = 0, 1$ and can be neglected for $l \geq 2$. So, the complex shifts remove the degeneration of the ns and np -levels, whereas other levels remain to be degenerated. A dependence of the complex

shifts on the principal quantum number can be obtained taking into account that the radius of annihilation region and of nuclear interaction is small as compare with a radius of the antiprotonic hydrogen atom. Radial wave function of antiprotonic atom this region should have the simple form $R_{nl}(r) \simeq C_{nl} \cdot r^l$. It allows to express the dependence the complex shifts ΔE_{nl} on n in terms of the coefficients C_{nl} :

$$\Delta E_{nl} = \Delta E_{l+1,l} |C_{nl}/C_{l+1,l}|^2 \quad (3)$$

The coefficients C_{nl} are estimated, as a rule, with the hydrogen-like wave functions that gives

$$\Gamma_{ns} = \Gamma_{1s}/n^3, \quad (4)$$

$$\Gamma_{np} = \Gamma_{2p} \cdot 32(n^2 - 1)/(3n^5), \quad (5)$$

and similarly for ϵ_{nl} . As the input data we have taken the following values for the energy shifts and widths [8]:

$$\Gamma_{1s}/2 = 561 \text{ eV}, \quad \epsilon_{1s} = 691 \text{ eV}, \quad \Gamma_{2p}/2 = 17 \text{ meV}, \quad \epsilon_{2p} = 0. \quad (6)$$

With these values we obtain from the Eqs. (4), (5) for the states with $n = 30$:

$$\Gamma_{ns}/2 = 20.78 \text{ meV}, \quad \epsilon_{ns} = 25.59 \text{ meV}, \quad \Gamma_{np}/2 = 0.00671 \text{ meV}, \quad \epsilon_{np} = 0. \quad (7)$$

Total hamiltonian H of the antiprotonic hydrogen atom in the external electric field can be written as a sum of two terms,

$$H = H_0 + V, \quad (8)$$

where H_0 is a diagonal matrix with elements

$$\langle nl|H_0|nl'\rangle = \delta_{ll'} (\delta_{l0}\Delta E_{ns} + \delta_{l1}\Delta E_{np}). \quad (9)$$

The matrix of interaction of the atom with the electric field E directed along Oz axis has the form:

$$\langle nlm|V|nl'm'\rangle = eEr_0(3/2)n\sqrt{n^2 - l_{>}^2} \langle l010|l'0\rangle \langle l'm'10|lm\rangle, \quad (10)$$

where $l_{>} = \max(l, l')$, the quantities like $\langle l010|l'0\rangle$ are Clebsh-Gordan coefficients, $r_0 = r_B/\mu$, $\mu = M_p/2$ is a reduced mass of the $(p - \bar{p})$ system in the units of electron mass, and r_B is the Bohr radius of ordinary hydrogen atom.

Eigenvalues of the non-hermitian matrix (8) give the complex energies of the $(p\bar{p})_n$ states in the static electric field. The real parts of these energies are shifts of the levels from the unperturbed values and the imaginary parts are the annihilation widths with account for a mixing of the states with different l due to electric field.

3 Results and Discussion

The formulation of the problem, as given in the previous section, supposes that a mixing of the states with different l at fixed n is important, whereas a mixing of the states with different n can be neglected. It means that our consideration has to be restricted by

the field less than a critical value, at which non-diagonal matrix element $\langle nl|V|n'l' \rangle$ is comparable with the distance between the levels with different n . It is easy to estimate that for the levels with $n \simeq 30$ this critical value of the field is $E_a \sim 10^9$ V/cm, that is far from real laboratory fields.

It should be noted that in the problem under consideration there are also two other critical values of the field. The first one is the field that mixes effectively ns and np states, and other one is that mixed np and nd states, the latter being degenerated with all other nl states for $l > 2$. With the above-mentioned values of strong interaction shifts and annihilation widths of the ns and np states, we estimate the corresponding critical fields as $E_b \sim 3.3 \cdot 10^6$ V/cm for a mixing of ns and np at $n = 30$, and $E_c \sim 2000$ V/cm for a mixing of np and nd states. We restrict our consideration by the fields up to 5000 V/cm, therefore only third critical value E_c of the field is covered by these calculations.

For the further discussion of the obtained results let us remember also, that the problem of Stark mixing in the degenerated (without initial shifts and widths) hydrogen-like system at the fixed n has the exact solution in the parabolic coordinates [9]. Eigenstates of a such system are labeled by parabolic quantum numbers n_1, n_2, m ($n_1 + n_2 + |m| + 1 = n$), the splitting of the levels is linear in electric field, whereas the coefficients $\langle nlm|n_1n_2m \rangle$ of mixing of the states with different l do not depend on the value of the field and can be expressed in terms of Clebsh-Gordan coefficients,

$$\langle nlm|n_1n_2m \rangle^2 = \langle k\nu_1k\nu_2|lm \rangle^2, \quad (11)$$

where $k = (n - 1)/2$, $\nu_1 = (n_1 - n_2 + m)/2$, $\nu_2 = m - \nu_1$. Therefore the admixture of ns -state to all parabolic states with $m = 0$ should be $\langle ns|nn_1 \rangle^2 = 1/n$, and the weights of the np -state in the parabolic states are $\langle np, m|nn_1, m \rangle^2 = 0.019 \div 0.025$ at $n = 30$. These relationships could describe the effect of external field on antiprotonic hydrogen at very high values of the electric field ($E_b < E < E_a$). However for the realistic values of $E < E_b$ the influence of the field can not be predicted without calculations.

The results of our calculations are shown on Figs. 1 - 4. On these figures the shifts $W(E)$ and widths $\Gamma(E)$ of the states are presented in the relative units of $|W_{\max}|$ and Γ_{\max} , which are the maximum values of these quantities in the considered interval of electric fields ($0 \leq E \leq 5000$ V/cm).

Different types of dependencies of the shifts on the value E of electric field are shown on Figs. 1 - 2 for several levels with $m = 0$. The levels are labeled by the indexes l that correspond to the quantum numbers of the states in the limit $E = 0$. With respect to the dependence of the shifts on E , the levels can be divided into two main types. The shifts are negative for $l \leq 14$ and positive for $l \geq 16$, and in the both cases they rise in absolute value with increasing of the electric field. Within the intermediate interval $l \in 13 - 16$ the shifts are very small and change the signs. It is seen from Fig. 2 that in this region the shifts are nonlinear in E and, moreover, they behave themselves for the first sight unusually, showing a resonance-like behaviour at $E \sim E_c$. However, the nature of a such behaviour is quite clear - this is a result of some crossing of the levels that takes place with a change of the values of electric field. For the levels with $|m| = 1$ we observe a similar behaviour with E .

The widths of the levels with $m = 0$ are shown on Figs. 3 - 4 depending on the value of electric field. For the states with $l \leq 13$ and $l \geq 16$ the widths grow with increasing of the electric field, however in the aforementioned interval $13 \leq l \leq 16$ the widths depend

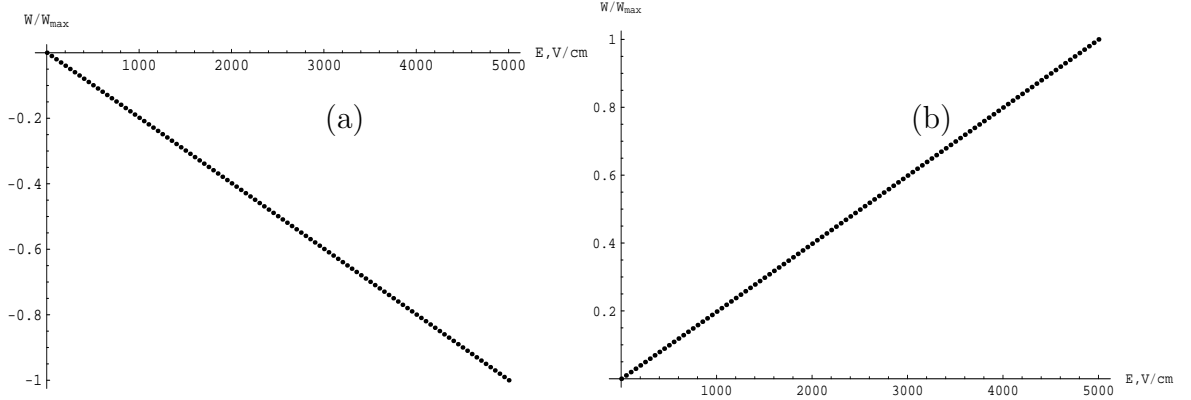


Figure 1: The dependence of the shifts of $(p\bar{p})$ states on electric field. Part (a): $l = 1$, $W_{\max} = 3.7 \cdot 10^{-5}$ eV; part (b): $l = 28$, $W_{\max} = 3.43 \cdot 10^{-5}$ eV.

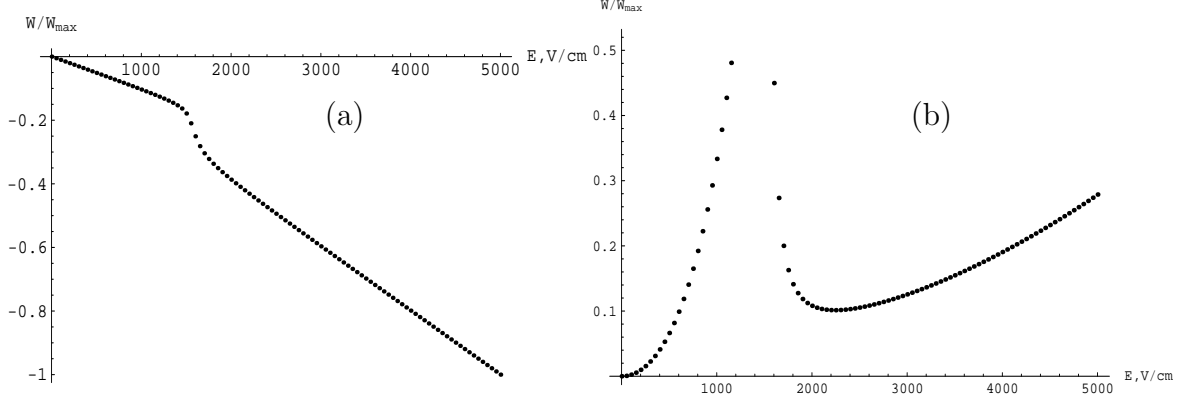


Figure 2: Resonance-like dependence of the shifts of $(p\bar{p})$ states on electric field. Part (a): $l = 14$, $W_{\max} = 2.62 \cdot 10^{-6}$ eV; part (b): $l = 15$, $W_{\max} = 1.95 \cdot 10^{-9}$ eV

on the field in a resonance-like manner. The reason of this phenomenon is the same as for the shifts, i.e. it is connected with the crossing of levels in the static electric field.

The shift and width of the ns state are practically unchanged at the considered values of E . It is related with the large values of the 'self' shift and width, or, in other words, with the fact that the value of E is small as compare with critical field E_b as estimated above. Thus the obtained values of 'induced' widths of all the states with $l \neq 0$ are due to the influence of np -states in the considered interval of electric fields. At larger electric fields the contribution of ns states would be a more essential.

4 Conclusion

In this paper we have made the analysis of the splitting and annihilation widths of the levels of antiprotonic hydrogen atom in the static external electric field. As compare with the theoretical case when all the levels are degenerated and the exact analytical solution of this problem exists, we meet here a more complicated situation. This complication

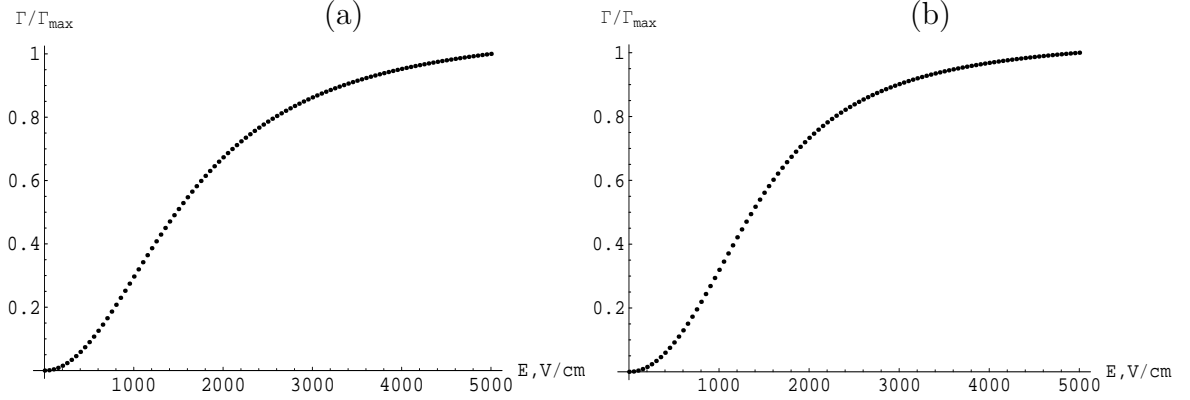


Figure 3: Dependence of the widths on the value of electric field. Part (a): $l = 1$; $\Gamma_{\max} = 3.42 \cdot 10^8$ 1/s; part (b): $l = 28$, $\Gamma_{\max} = 4.52 \cdot 10^8$ 1/s.

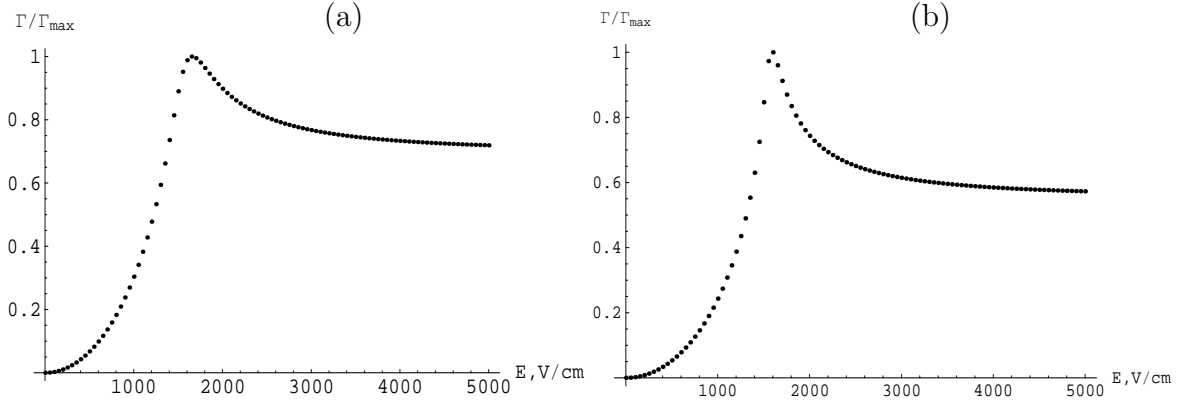


Figure 4: Resonance-like dependence of the widths on the value of electric field. Part (a): $l = 14$, $\Gamma_{\max} = 1.51 \cdot 10^9$ 1/s; part (b): $l = 16$, $\Gamma_{\max} = 1.52 \cdot 10^9$ 1/s.

is related with the strong interaction shifts and annihilation widths of ns and np states participated in the mixing of nl levels of the antiprotonic atom. The results of the paper can be formulated as follows:

a) due to mixing of different nl states ($l > 1$) with np states at the value of electric field $E \sim 1000$ V/cm, all the states acquire the widths of order $10^8 \div 10^9$ 1/s. The relevant shifts for the most part of the states are linear in E and have the order of value 10^{-5} eV.

b) Contribution of the ns state into the widths of other levels is negligible at the considered values of electric field ($E \leq 5000$ V/cm), because the ns state is very distant in complex energy from all other levels and can be admixed to these states at $E \sim 3 \cdot 10^6$ V/cm. As a result, the main contribution to the widths of the mixed levels gives np state.

c) For the states that are close to the point, where the sign of the shift is changed, the shifts and widths of exact states have a resonance-like dependence on the value of electric field. The 'resonance' value of electric field is of the same order as critical value E_c , which provides strong mixing of the np state with other states ($l \geq 2$). This phenomenon is due to a crossing of the levels at some value of the electric field.

Acknowledgements

This investigation was supported by Russian Foundation for Basic Research as a part of the project 03-02-16616. Authors thank to N.P. Yudin for the useful discussions. One of the authors (G.K.) thanks to Y. Yamazaki for attracting our interest to the considered problem.

References

- [1] T.Yamazaki, N.Morita, R.S.Hayano, E.Widmann and J.Eades, Physics Reports 366 (2002) 183.
- [2] A.Bertin, M.Bruschi, M.Capponi et al., Phys. Rev. A 54 (1996) 5441; E.Lodi Rizzini, A.Bianconi, G.Bonomi et al., Phys.Lett. B 507 (2001) 19.
- [3] ASACUSA collaboration. Status report CERN/SPSC 2002-002, SPSC/M-674, Jan. 2002.
- [4] G.Ya.Korenman, Hyperfine Interaction, 101/102 (1996) 81.
- [5] J.S.Cohen, Phys.Rev. A 56 (1997) 3583.
- [6] K.Omidvar, Atomic Data and Nucl. Data Tables 28 (1983) 1.
- [7] M.Leon and H.A.Bethe, Phys. Rev. 127 (1962) 636.
- [8] T.P.Terada and R.S.Hayano, Phys. Rev. C 55 (1996) 73.
- [9] L.D.Landau and E.M.Lifshits, Quantum Mechanics, Fizmatgiz, Moscow, 1963, § 77.